International Workshop on Climate Downscaling Studies

Integrated Research Program for Advancing Climate Models (TOUGOU)
Theme C: Integrated Climate Change Projection

Estimating Design Rainfalls Using Dynamical Downscaling Data

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Outline

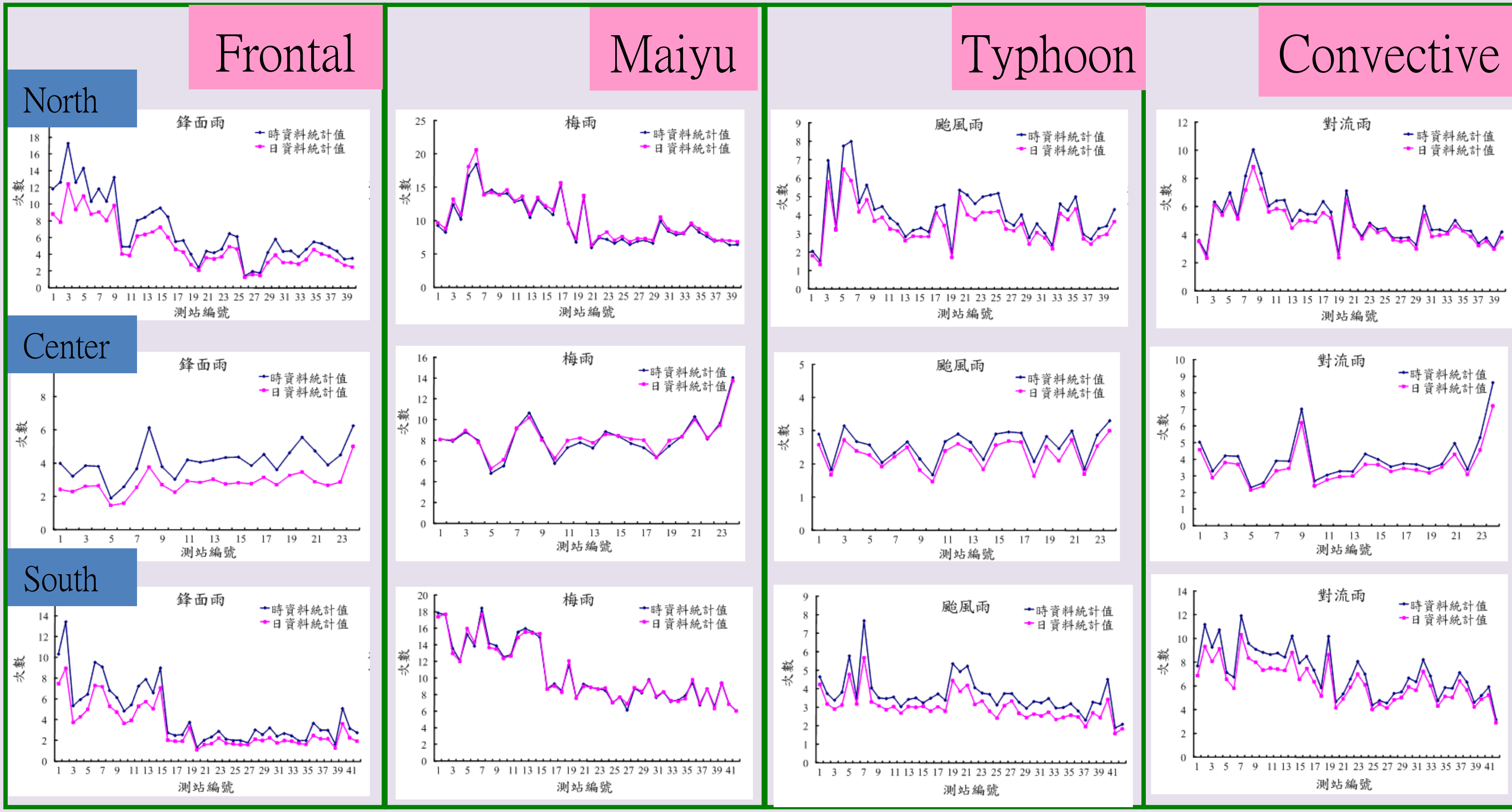
- **Introduction**
- **GEV modeling of the extreme values**
- **Mixture distribution of order statistics**
- **Demonstration by stochastic simulation**
- **Demonstration using observed rainfall data in Taiwan**
- **Conclusions**

Introduction

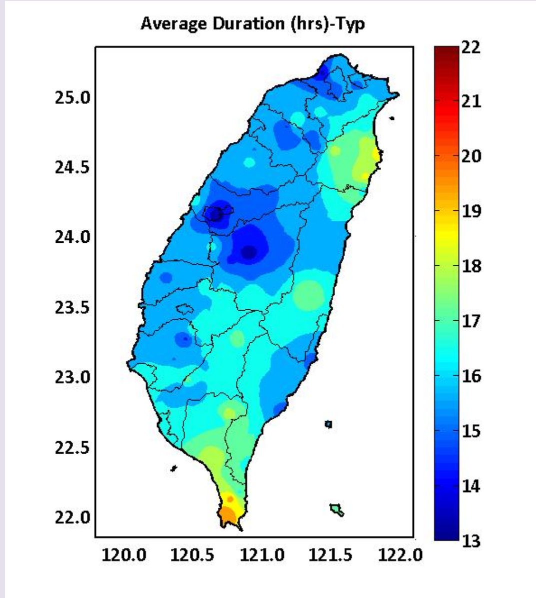
- **Design rainfalls**
  - **Design rainfalls represent rainfall depths of specific storm durations (e.g., 1, 2, 6, 12, 24, 48, 72 hours) which, on average, occur once in every T (5, 20, 100, 200, 1000) years.**
  - **Design rainfalls are essential for hydrological modeling and water resources planning and engineering design.**
  - **Estimation of design rainfalls under climate change requires downscaling GCM rainfall outputs to hourly (or even sub-hourly) scale.**

- **Previous work – Stochastic Storm Rainfall Simulation Model (SSRSM)**
  - **Annual count of storm occurrences (Poisson distribution)**
  - **Bivariate distribution of storm duration and event-total rainfall depth (Non-Gaussian bivariate distribution)**
  - **Temporal variation of hourly rainfalls of individual storms (Bivariate truncated-gamma distribution + Markov process)**
  - **Meiyu (MCS), summer convective storms, typhoons, and winter frontal rainfalls are treated separately.**

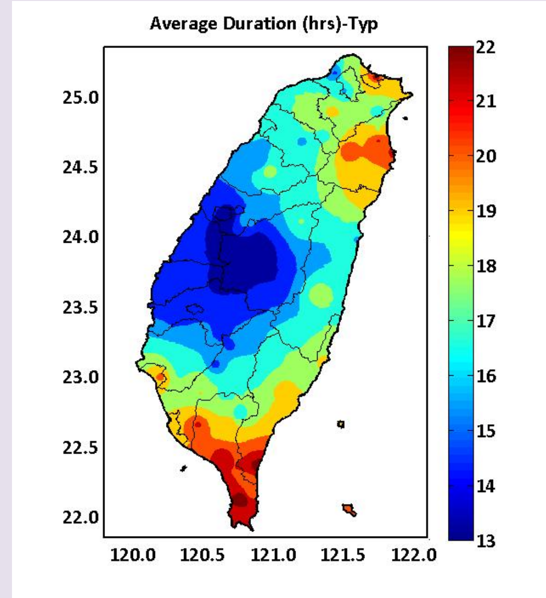
Annual counts of storm events estimated by ANN



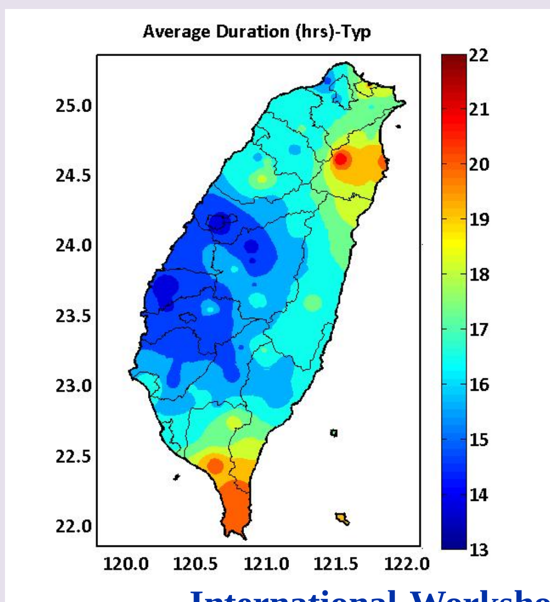
Storm characteristics (average duration of typhoon)



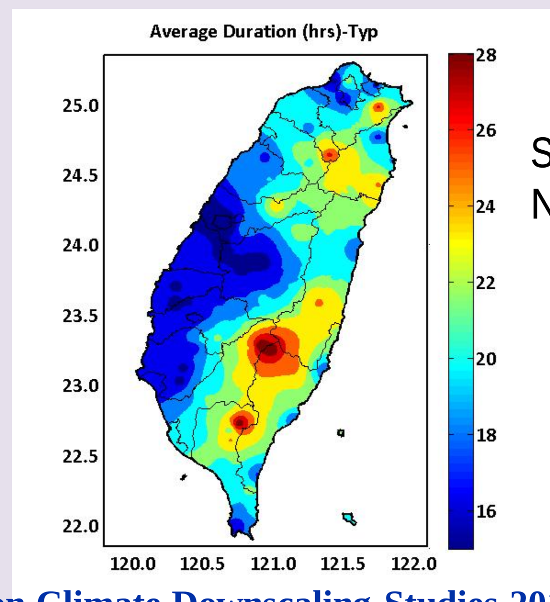
Gauge observations



MRI (1979 - 2003)



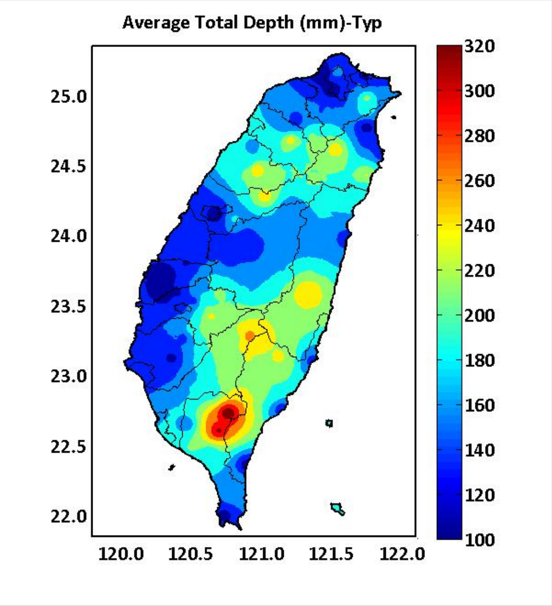
MRI (2015 – 2039)



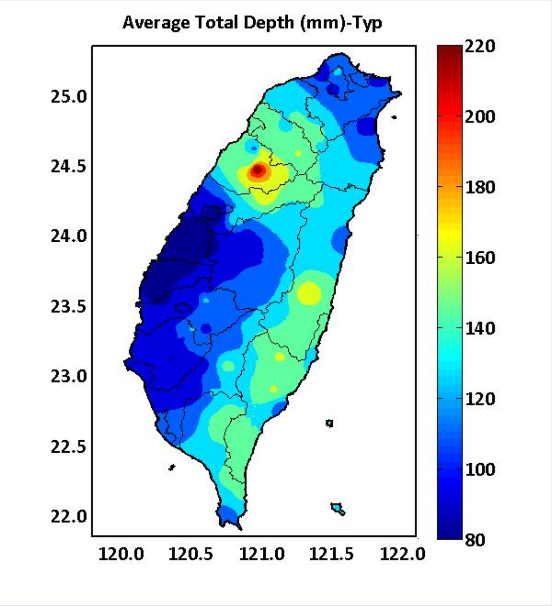
MRI (2075 - 2099)

Source:
NCDR, Taiwan

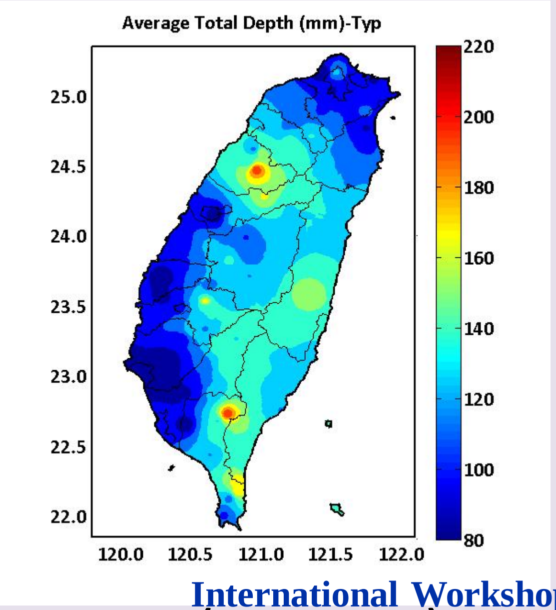
Storm characteristics (average event-total rainfalls of typhoon)



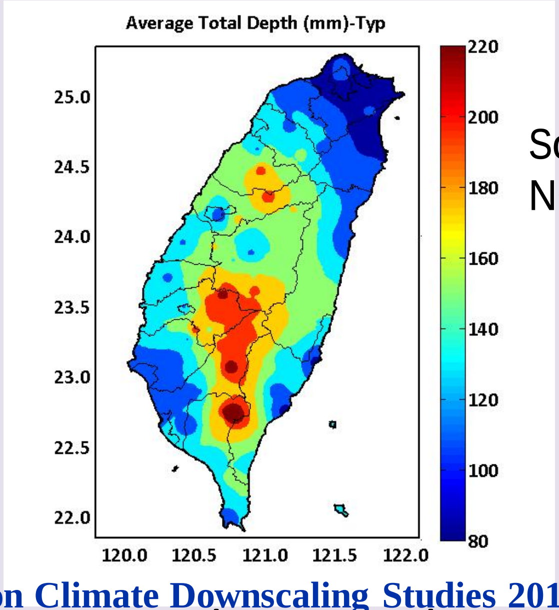
Gauge observations



MRI (1979 - 2003)



MRI (2015 – 2039)



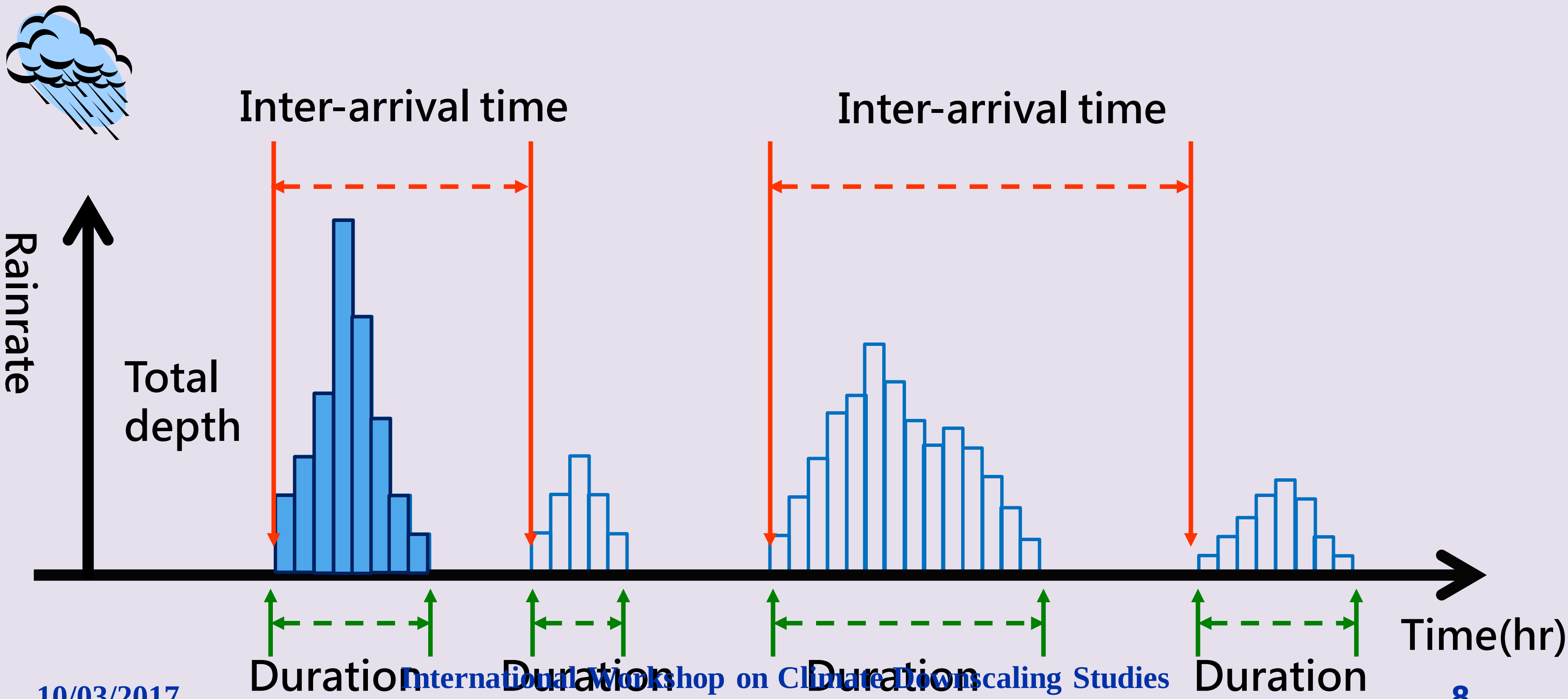
MRI (2075 - 2099)

Source:
NCDR, Taiwan

Stochastic storm rainfall process

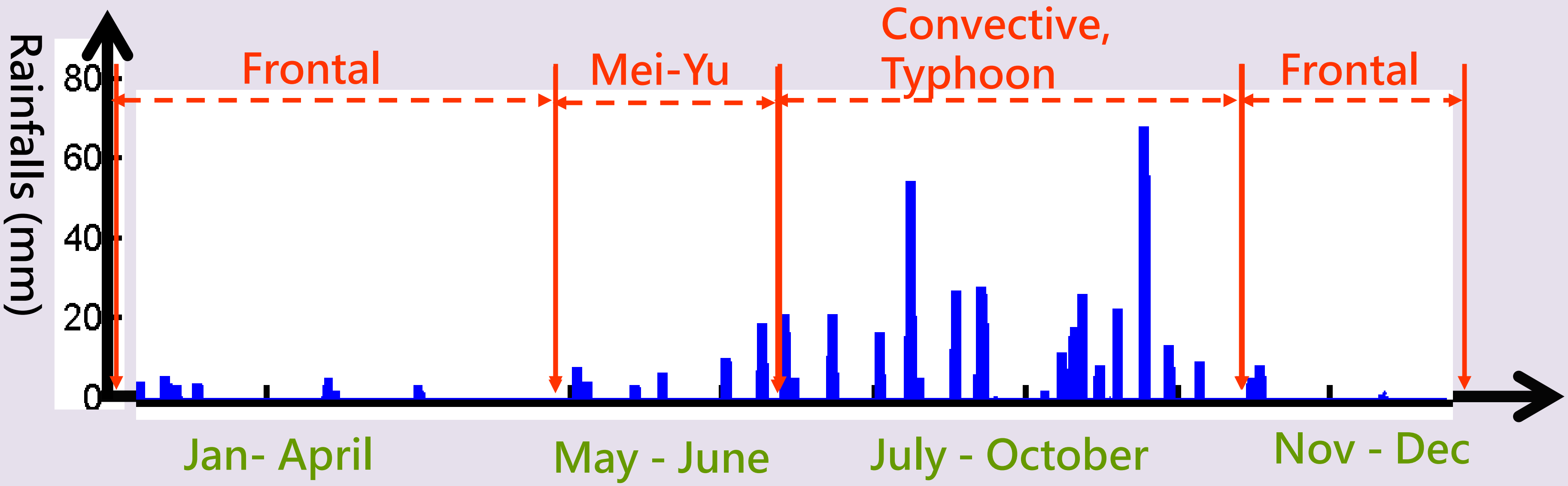
Storm characteristics

- Duration
- Event-total depth
- Inter-arrival(or inter-event) time
- Time variation of rain-rates

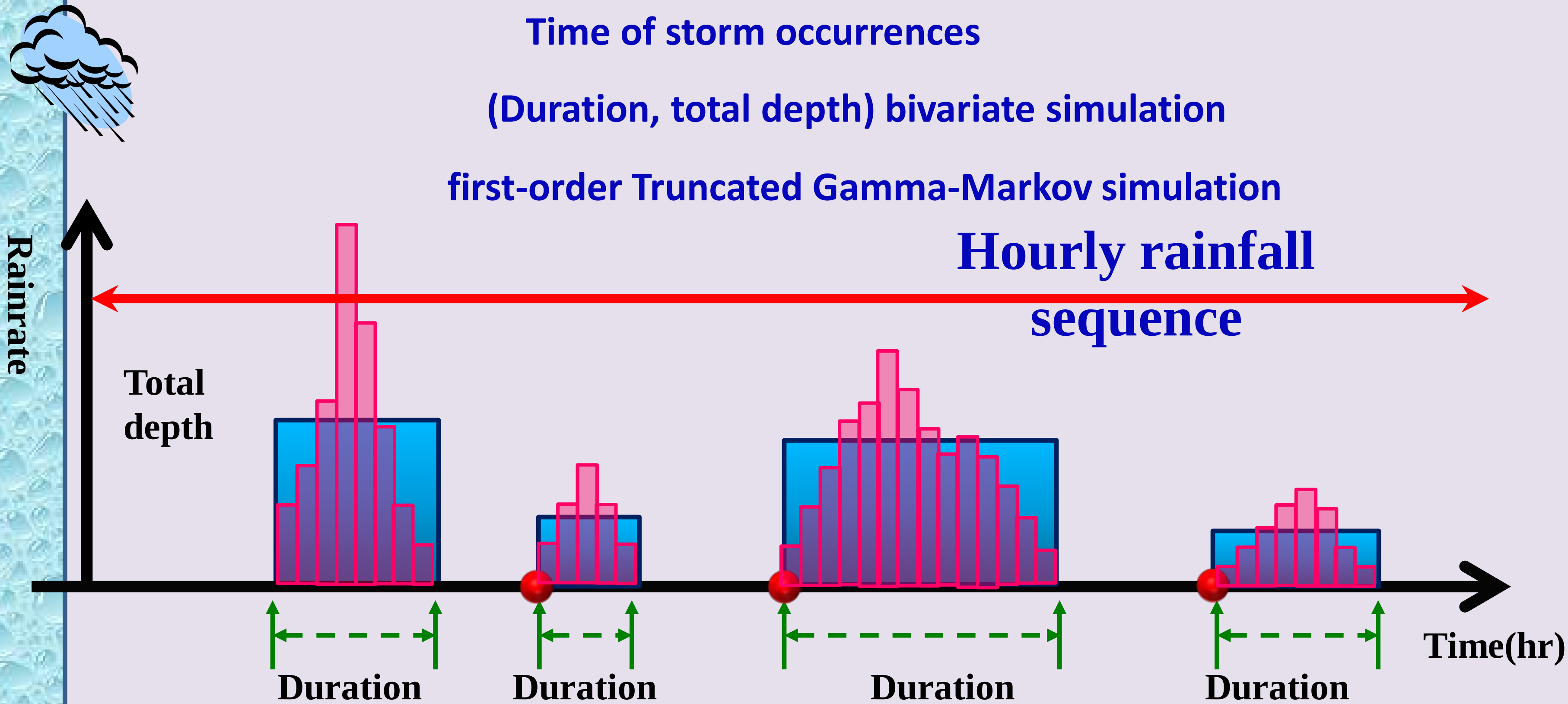


Season-specific storm characteristics

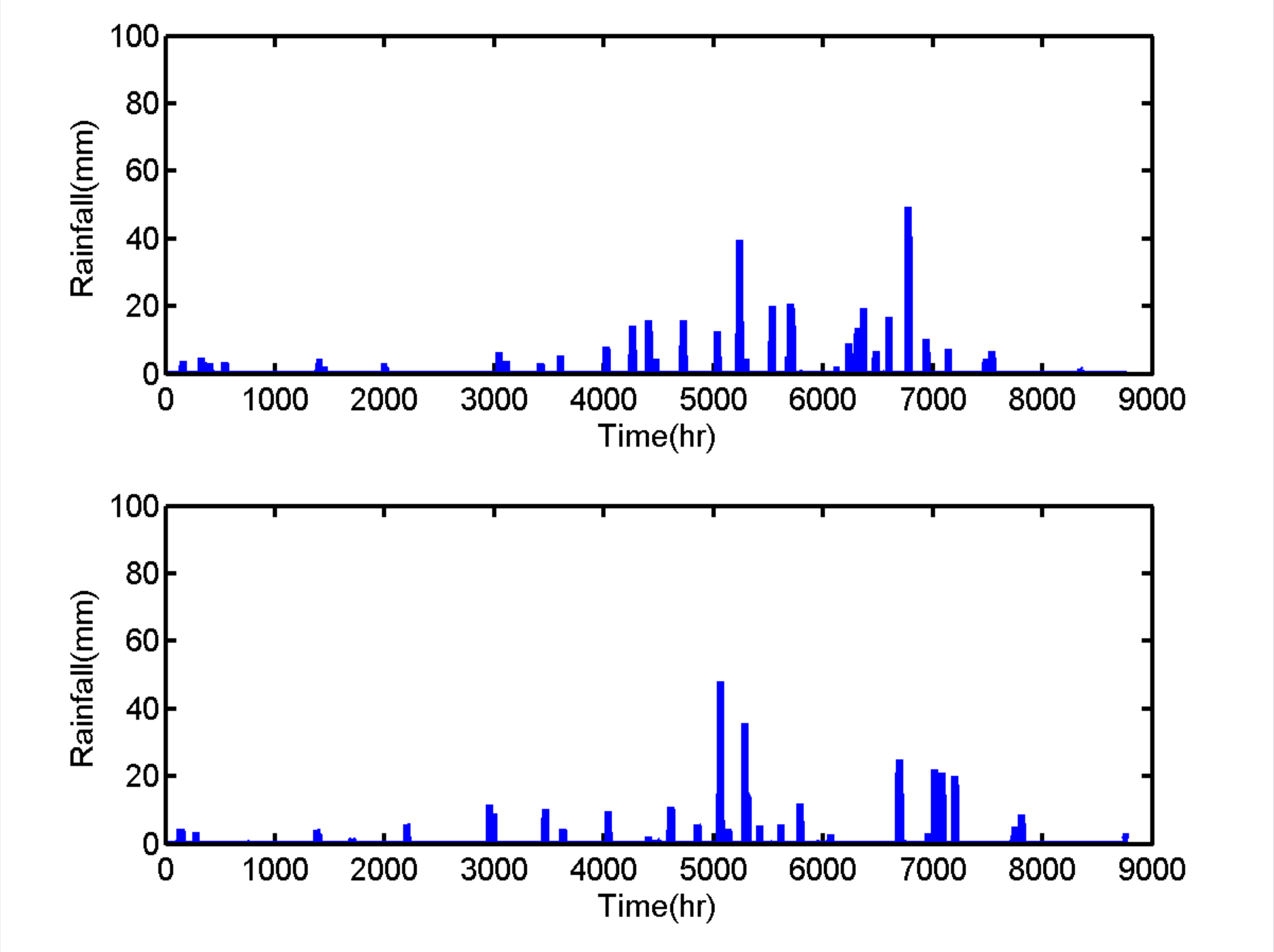
Storm type	Period
Frontal	Nov - April
Mei-Yu	May - June
Convective	July - October
Typhoon	July - October



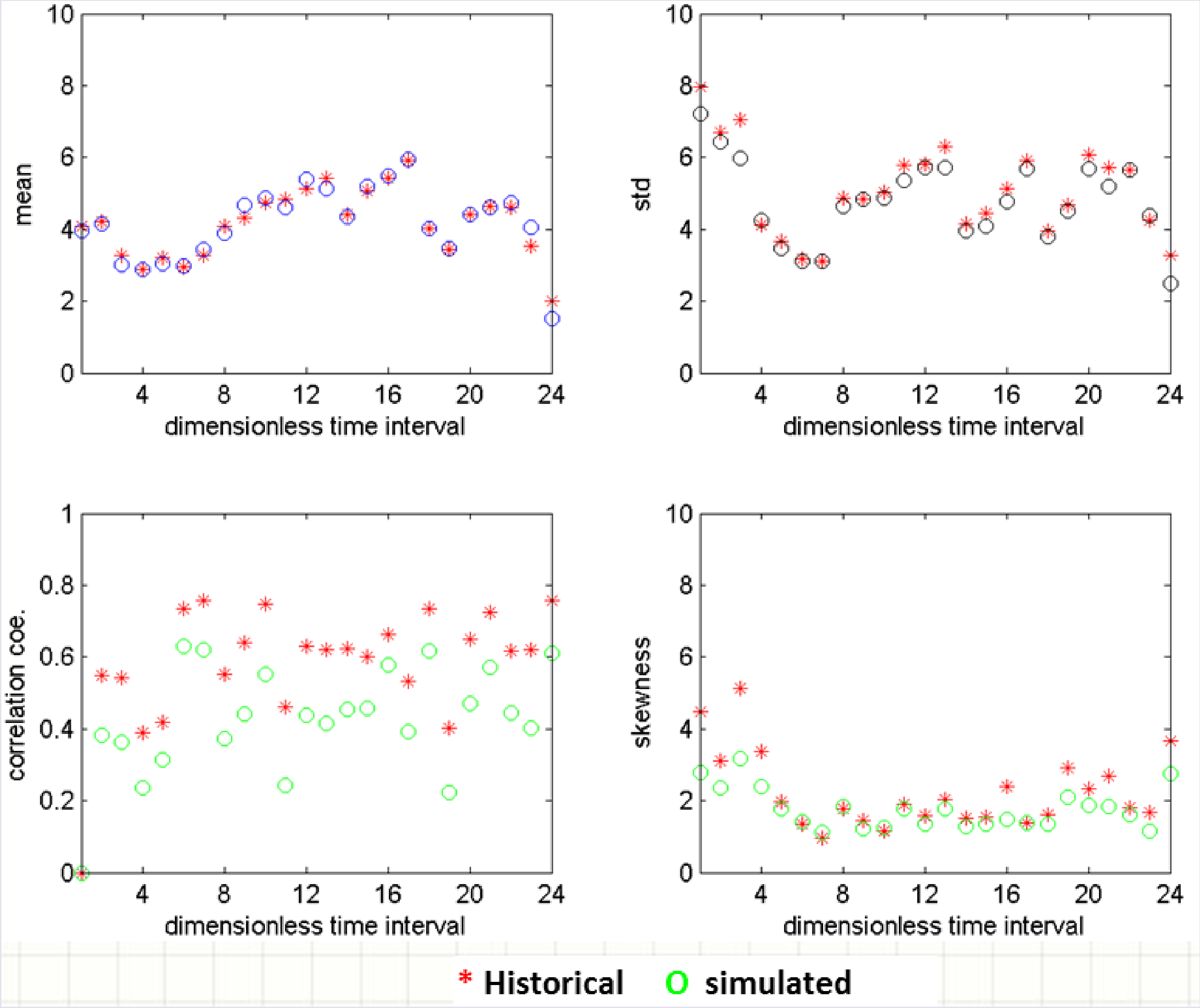
Each simulation run yields an annual sequence of hourly rainfalls. 500 runs were generated for each rainfall station.



Examples of hourly rainfall sequence (Kaoshiung)

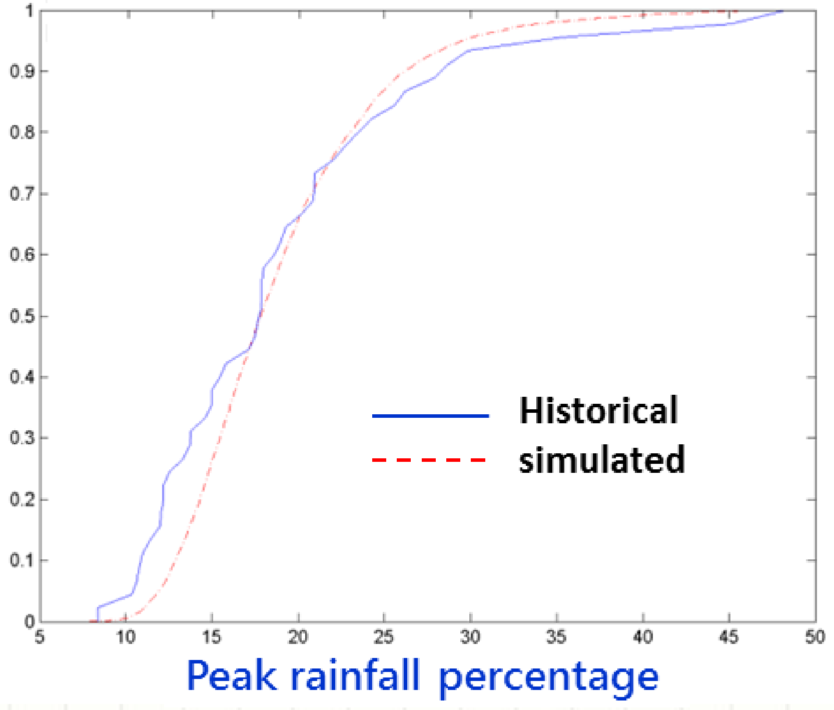


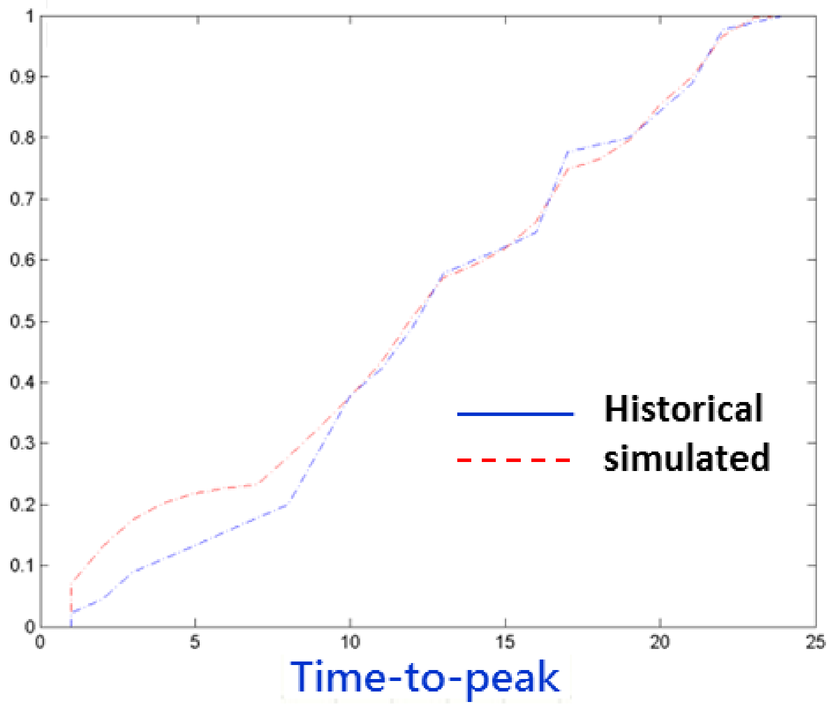
Hyetograph Simulation results (Typhoons)



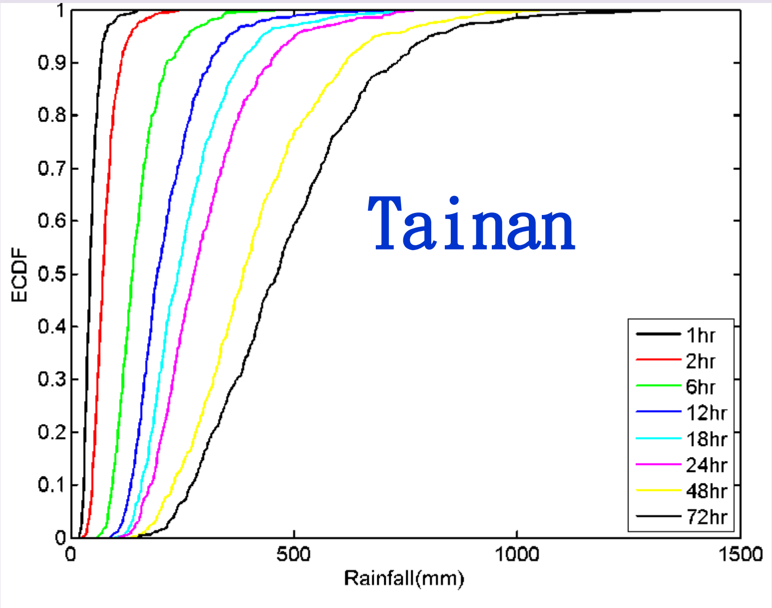
Time-to-peak and peak rainfall percentage (Typhoons)

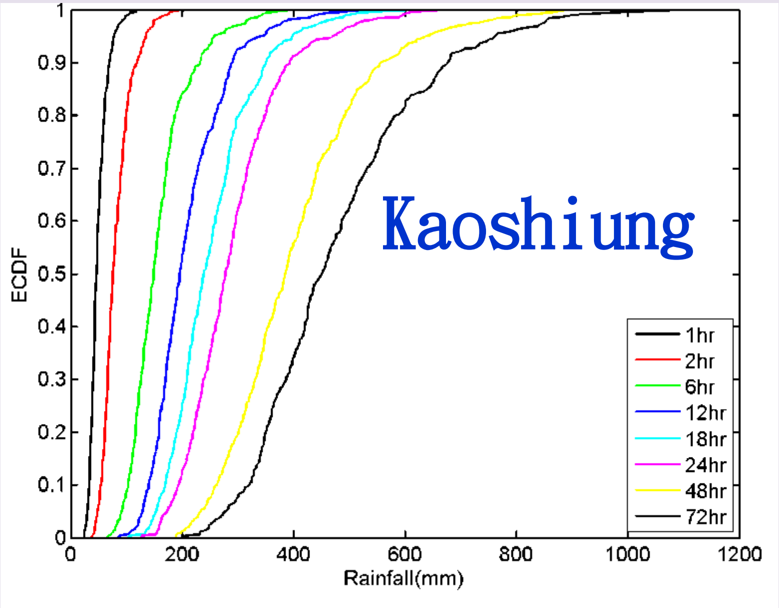
- **Empirical cumulative distribution functions**





Impact on design storm depths





(Projection period: 2020-2039)

Introduction

- **Most rainfall frequency analyses were conducted using annual maximum series. However, for stations with short record length, i.e. small sample size, (for example, less than 30 years) and with presence of outliers, results of rainfall frequency analysis will be less reliable.**
- **Annual maximum rainfalls of longer durations (> 12 hours) in Taiwan were mostly produced by typhoon or meiyu events.**

- **We propose using event-maximum rainfalls for rainfall frequency analysis.**
- **By using the event-maximum rainfalls (of various design durations), the sample size can be increased.**
- **The annual count of events (in our case, typhoons) can vary from one year to another and can be considered as a Poisson random variable.**

- **The average annual count of typhoons in Taiwan is roughly 3.4 (depending on locations). Mean value of the Poisson distribution.**
- **The basic idea is** [...] **frequency analys**[...] **distribution of t-hr event-maximum rainfalls and the Poisson distribution which characterizes the occurrence of typhoons. [A mixture distribution]**

Can be readily provided by high resolution dynamic downscaling data.

- **For stations whose annual maximum rainfalls were produced by different storm types (typhoons and meiyu events), a storm-type mixture of mixture distributions needs to be considered.**

GEV modeling of the extreme values

Extremal Type Theorem

Type I: $G_1(x) = \exp(-\exp(-x)), -\infty < x < \infty$

Type II: $$G_2(x) = \begin{cases} 0 & if \quad x \leq 0, \\ \exp(-x^{-\alpha}) & if \quad x > 0, \ \alpha > 0 \ . \end{cases}$$

Type III: $$G_3(x) = \begin{cases} \exp(-(-x)^{\alpha}) & if \quad x \leq 0, \\ 0 & if \quad x > 0, \ \alpha > 0 \ . \end{cases}$$

$$G_\eta(x) = \begin{cases} exp\{-(1 + \eta[\frac{x-\mu}{\sigma}])^{-1/\eta}\} & if \quad \eta \neq 0 \\ exp\{-exp(-[\frac{x-\mu}{\sigma}])\} & if \quad \eta = 0 \end{cases}$$

(Extremal types theorem). *Let* (X_n) *be independent with distribution function* F *and let* $X_{(n)} = \max_{1 \le i \le n} X_{(i)}$. *If there exist constants* $a_n > 0$ *and* b_n *and a non-degenerate distribution function* G *such that*

$$\mathbb{P}\Big(\frac{X_{(n)} - b_n}{a_n} \le x\Big) \xrightarrow{d} G(x),$$

then G *must be of the same type as one of the three* extreme value *classes below:*

Type I *(Fréchet):* $G_{1,\alpha}(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \exp(-x^{-\alpha}) & \text{if } x > 0 \end{cases}$ *for some* $\alpha > 0$

Type II *(Negative Weibull):* $G_{2,\alpha}(x) = \begin{cases} \exp\{-(-x)^{\alpha}\} & \text{if } x < 0 \\ 1 & \text{if } x \ge 0 \end{cases}$ *for some* $\alpha > 0$

Type III *(Gumbel):* $G_3(x) = \exp(-e^{-x})$ *for* $x \in \mathbb{R}$.

$$G_\eta(x) = \begin{cases} exp\{-(1+\eta[\frac{x-\mu}{\sigma}])^{-1/\eta}\} & if \quad \eta \ne 0 \\ exp\{-exp(-[\frac{x-\mu}{\sigma}])\} & if \quad \eta = 0 \end{cases}$$

Mixture distribution of order statistics for annual maximum rainfalls

- **Order statistic**
  - **Given a random sample of size n ($X_1, X_2, \cdots, X_n$), the max order statistic $Y_n = \max(X_1, X_2, \cdots, X_n)$ satisfies $F_{Y_n}(y) = [F_X(y)]^n$**
  - **Annual maximum rainfalls can be considered as the maximum order statistic of individual years.**
  - **However, the annual count of events has a Poisson distribution. Thus, annual maximum rainfalls of different years have different probability distributions.**

$$F_{Y_{n_1}}(y) = [F_X(y)]^{n_1} \neq F_{Y_{n_2}}(y) = [F_X(y)]^{n_2}$$

– **Mixture distribution modeling. Annual count of events (*k*) has a Poisson distribution.**

$$F_W(w) = \sum_{k=0}^{\infty} [F_{Y_k}(w)] p(k) = \sum_{k=0}^{\infty} \left[F_X(w)\right]^k p(k)$$

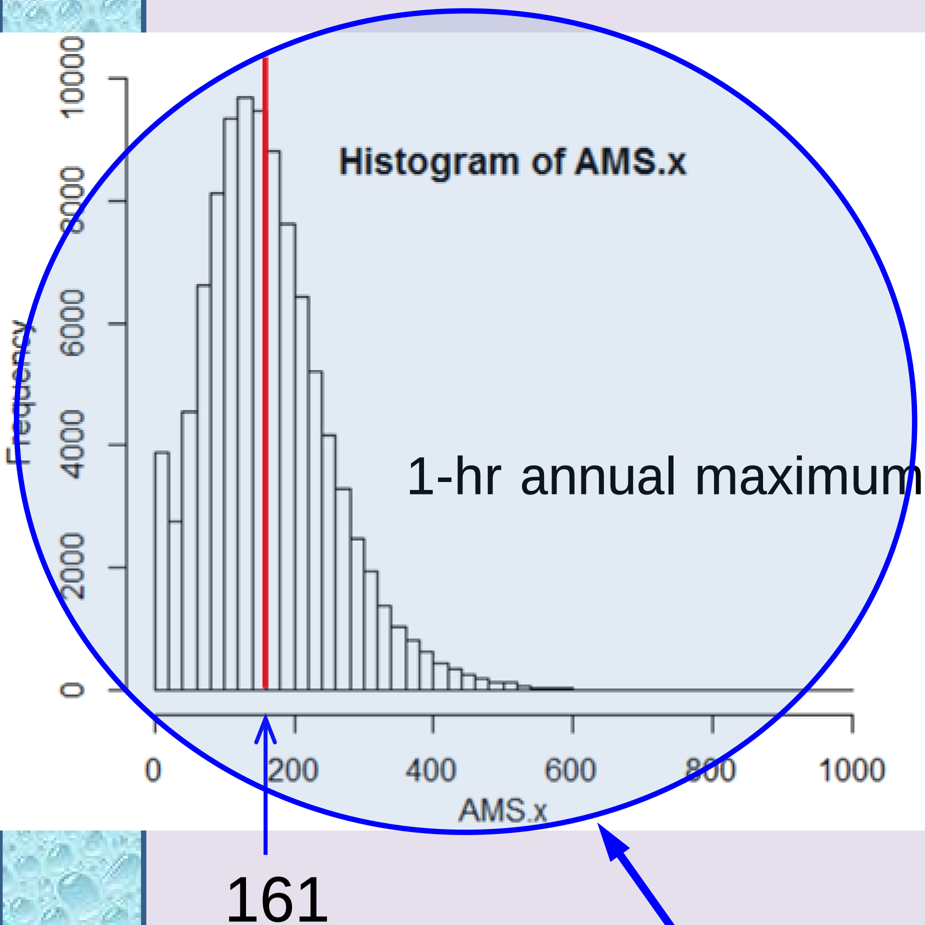
- **Three approaches (for calculation of T-yr rainfall, X_T)**
  - **Derive the probability distribution of annual maximum rainfalls from the event-maximum rainfalls and then find the 1-(1/T) quantitle of the annual maximum distribution. [Approach 1 - Mixture distribution approach]**
  - **Calculate X_T directly from the probability distribution of event-maximum rainfalls [Approach 2 - Adjusted exceedance probability approach]**

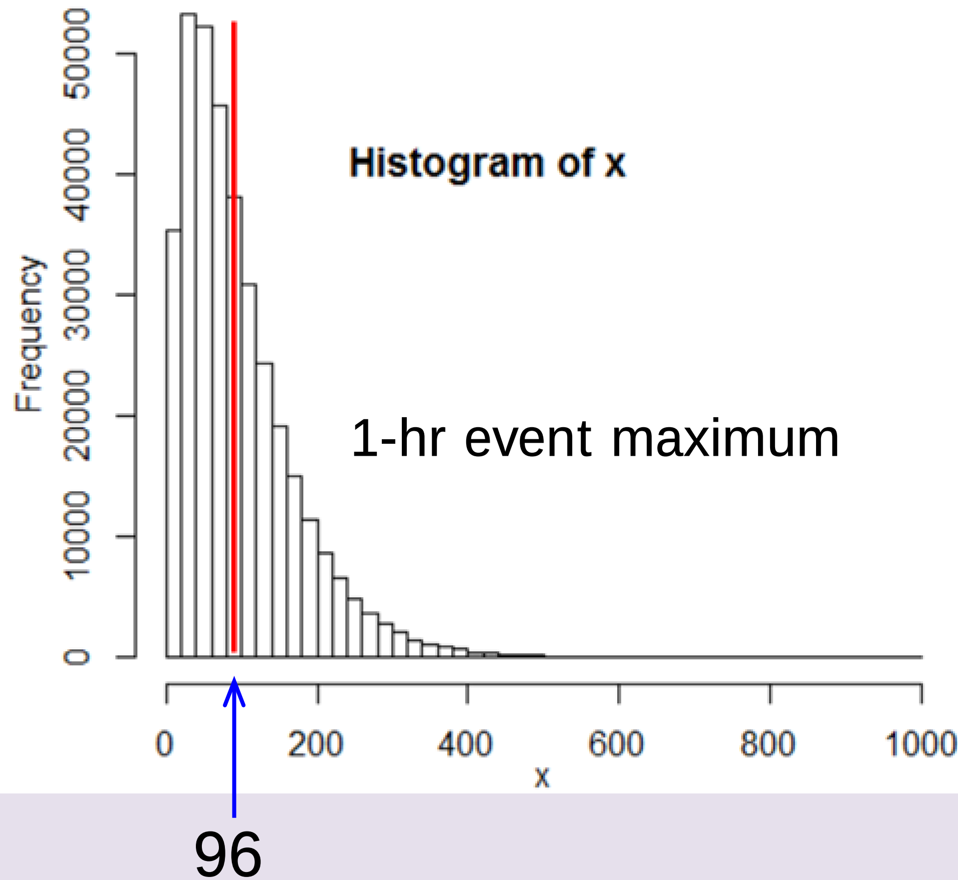
$$p_E = \frac{1}{T_E \cdot E(M)}$$

– **Simulate many years (N = 10,000) of typhoon occurrences and event-maximum rainfalls. Extract N annual maximum rainfalls from the simulation results and find X_T. [Approach 3 - Simulation approach]**

Demonstration by stochastic simulation

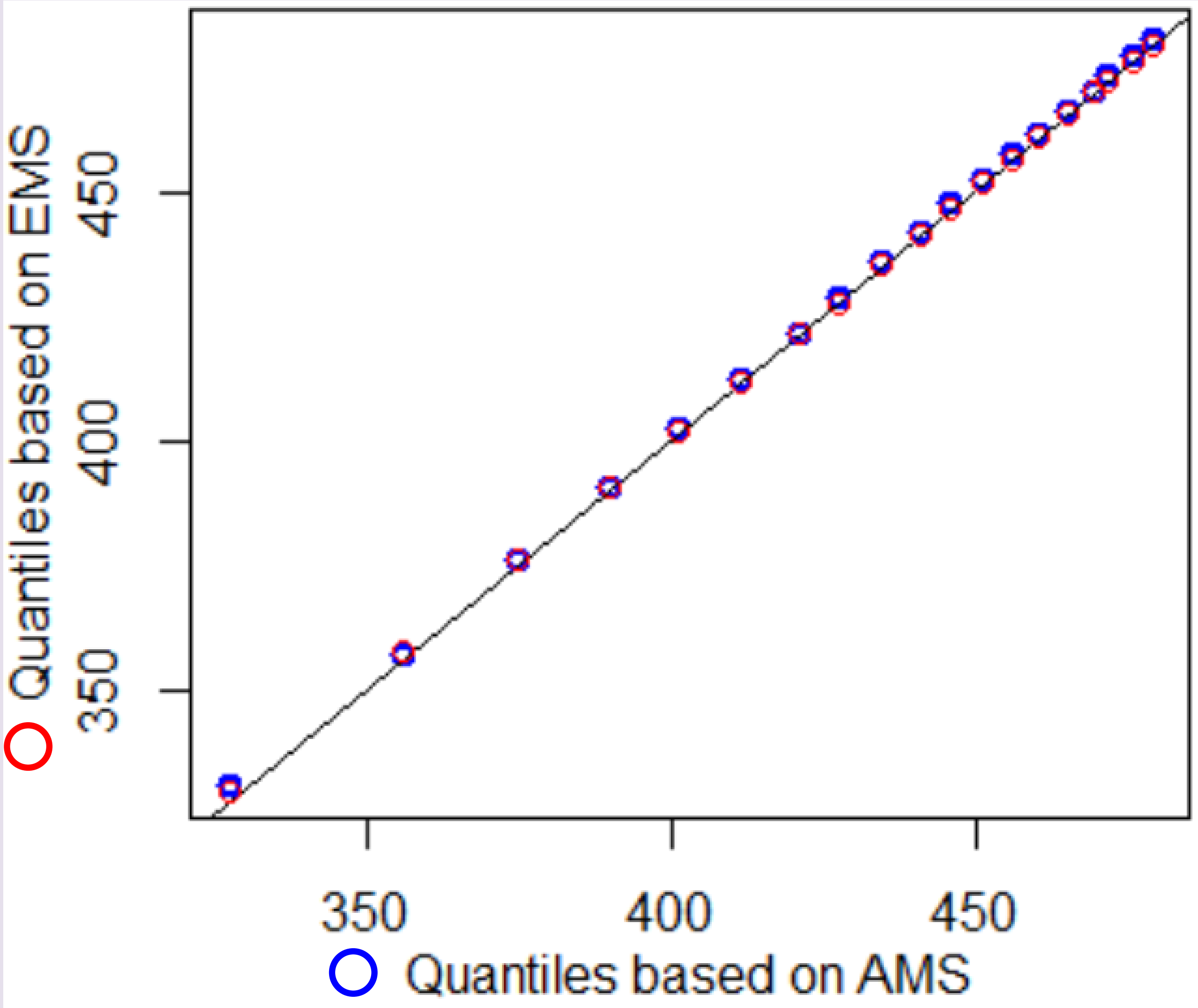
- Annual count of events has a Poisson distribution (mean = 3.6)
- 1-hour event-maximum rainfall has a gamma distribution with mean = 96 mm and stdev = 76 mm. (scale = 60, shape = 1.6)
- We simulated 100,000 years of typhoon occurrences and event-maximum rainfalls using R.



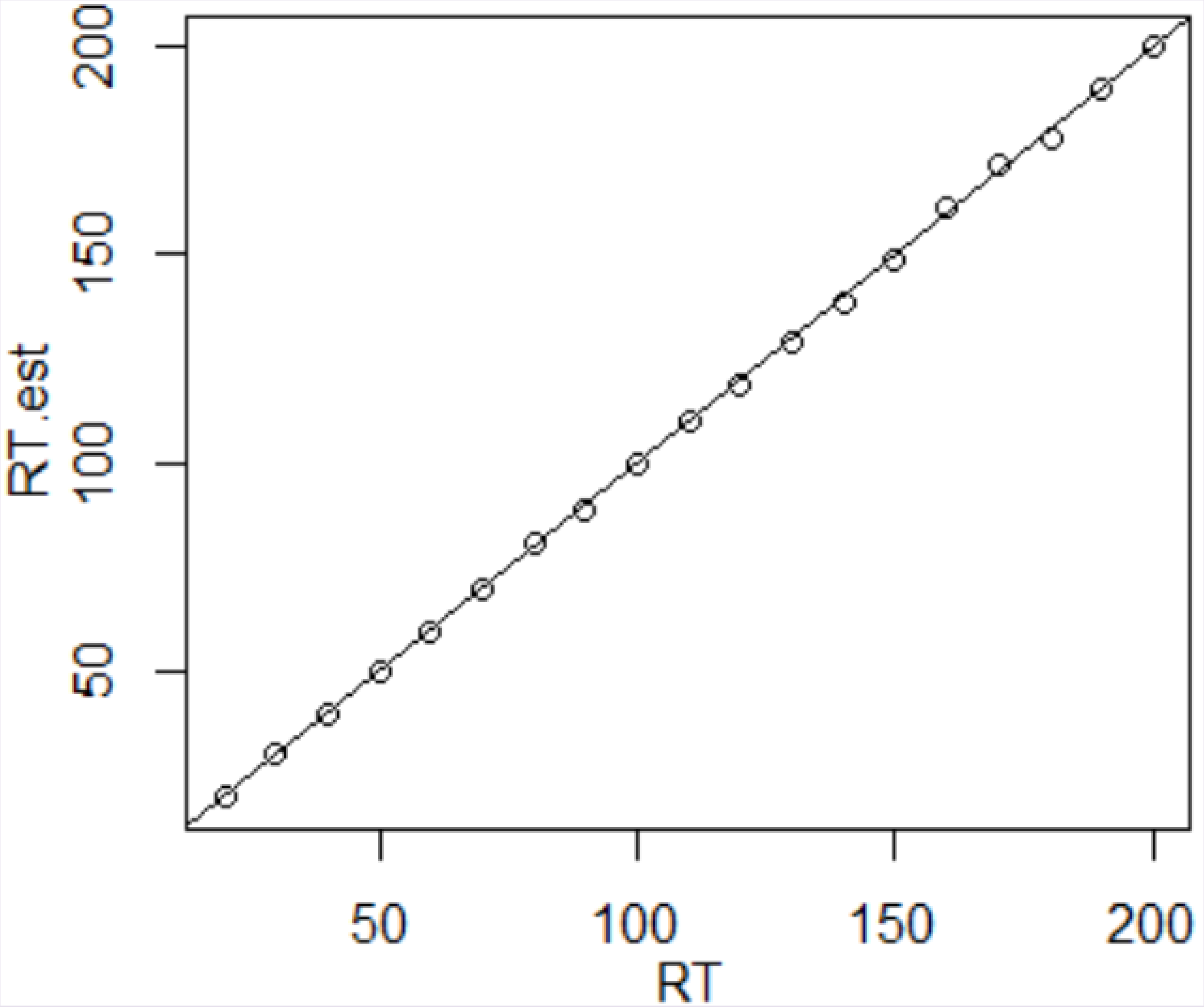


Can be considered as the theoretical distribution of annual maximam rainfalls

- **Rainfalls of various return periods (in years) (seq(20,200,10)) [Approaches 2 and 3]**



- **Comparison between Approaches 1 and 3**



- **The above simulation results demonstrate that annual maximum rainfalls are asymptotically and theoretically a mixture distribution of order statistics.**
- **Practically, dynamic downscaling data are available for 25 or 30-year period.**

Demonstration using observed rainfall data in Taiwan (limited record length)

- **44 years of event-maximum rainfalls of typhoons (頭汴坑) in Taiwan.**

AMS-based

Return period in years

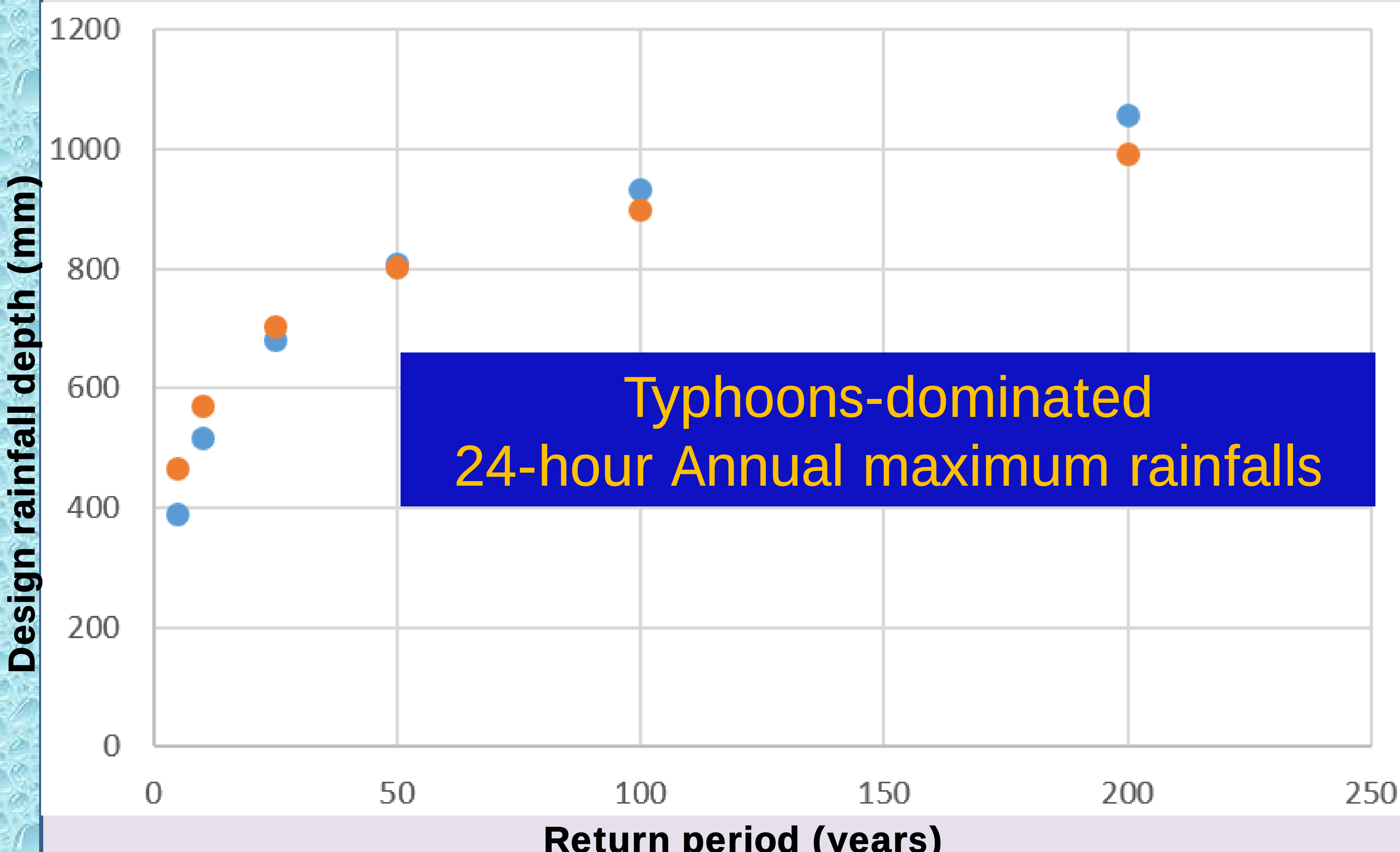
Duration	5	10	25	50	100	200
1	83	110	150	182	215	248
[illegible]	[illegible]	[illegible]	[illegible]	315	385	457
[illegible]	[illegible]	[illegible]	[illegible]	377	455	535
6	218	302	422	517	613	711
12	300	409	558	672	788	904
24	389	515	681	806	931	1056
48	460	600	782	918	1054	1189
72	495	640	827	967	1106	1245

Typhoons-dominated 24-hour Annual maximum rainfalls

Mixture distribution – Poisson distribution of annual count of typhoons

Duration	Return period in years 5	10	25	50	100	200
1	69	82	98	110	122	133
[illegible]	[illegible]	[illegible]	[illegible]	179	198	217
[illegible]	[illegible]	[illegible]	[illegible]	232	256	279
6	231	278	335	378	419	460
12	347	420	509	575	639	703
24	465	571	703	801	897	992
48	528	656	815	933	1050	1166
72	529	657	817	936	1053	1169

Typhoons-dominated 24-hour Annual maximum rainfalls



Design rainfall depth (mm)
1200
1000
800
600
400
200
0
0
50
100
150
200
250
Return period (years)
Typhoons-dominated
24-hour Annual maximum rainfalls

Demonstration using observed rainfall data in Taiwan (limited record length)

- **42 years of event-maximum rainfalls of typhoons (Jia-Yi) in Taiwan.**

AMS-based

Return period in years

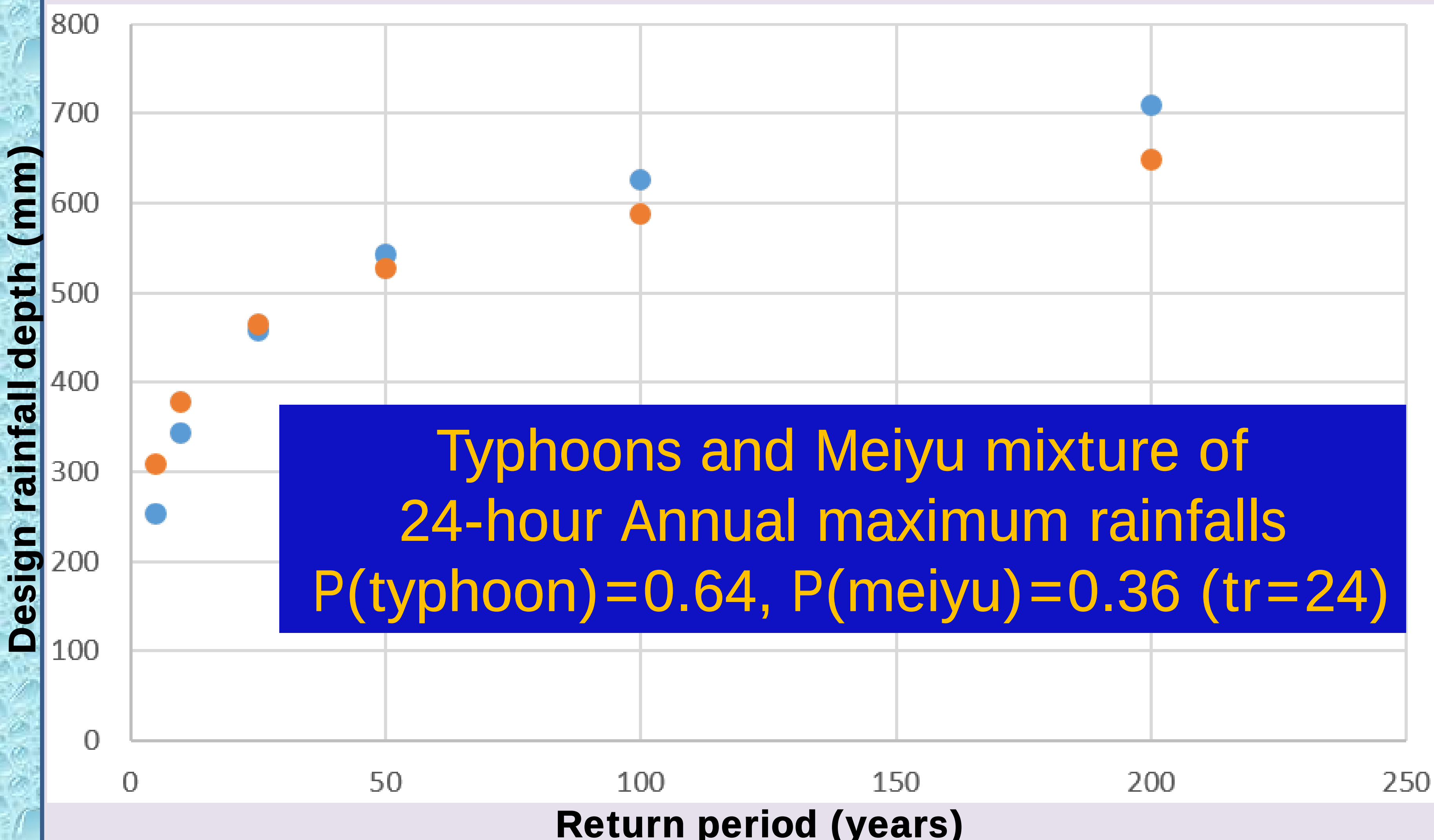
Duration	5	10	25	50	100	200
1	75.39092	94.09747	119.5949	139.2423	159.0994	179.1133
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]1858
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]9222
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]9471
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]4961
24	308.544	377.76	463.9536	526.6731	587.9276	648.0735
48	379.5444	464.9235	573.0741	652.68	730.9862	808.3108
72	417.6516	506.6674	618.9174	701.2927	782.1723	861.9217

Typhoons and Meiyu mixture of
24-hour annual maximum rainfalls
P(typhoon)=0.64, P(meiyu)=0.36 (tr=24)

- **A mixture of mixture distributions of typhoons and meiyu event-maximum rainfalls.**

Duration	5	10	25	50	100	200
1	55	70	92	109	126	144
2	[illegible]	[illegible]	[illegible]	[illegible]	194	221
3	[illegible]	[illegible]	[illegible]	[illegible]	229	260
6	[illegible]	[illegible]	[illegible]	[illegible]	314	354
12	[illegible]	[illegible]	[illegible]	[illegible]	454	512
24	253	344	458	542	626	709
48	281	392	535	641	747	852
72	282	394	537	644	749	855

Typhoons and Meiyu mixture of 24-hour Annual maximum rainfalls



Typhoons and Meiyu mixture of
24-hour Annual maximum rainfalls
P(typhoon)=0.64, P(meiyu)=0.36 (tr=24)

Conclusions

- **We demonstrate that the annual maximum rainfalls in Taiwan (which are produced by typhoons or meiyu events) can be characterized by a mixture distribution.**
- **For stations with short record length, using event-maximum rainfalls for rainfall frequency analysis can provide good estimates of design rainfalls.**
- **By adopting the proposed approach, design rainfalls of the projection period can be derived from high resolution dynamic downscaling data.**

Thanks for your attention.

24 GCMs → Projections in coarse spatial and time scales. (200 – 300 km; monthly) → RCM → Projections in finer spatial scale. (5km; monthly)

From GCM outputs to design storm depths – a problem of scale mismatch (both temporal and spatial)

Design rainfall depths

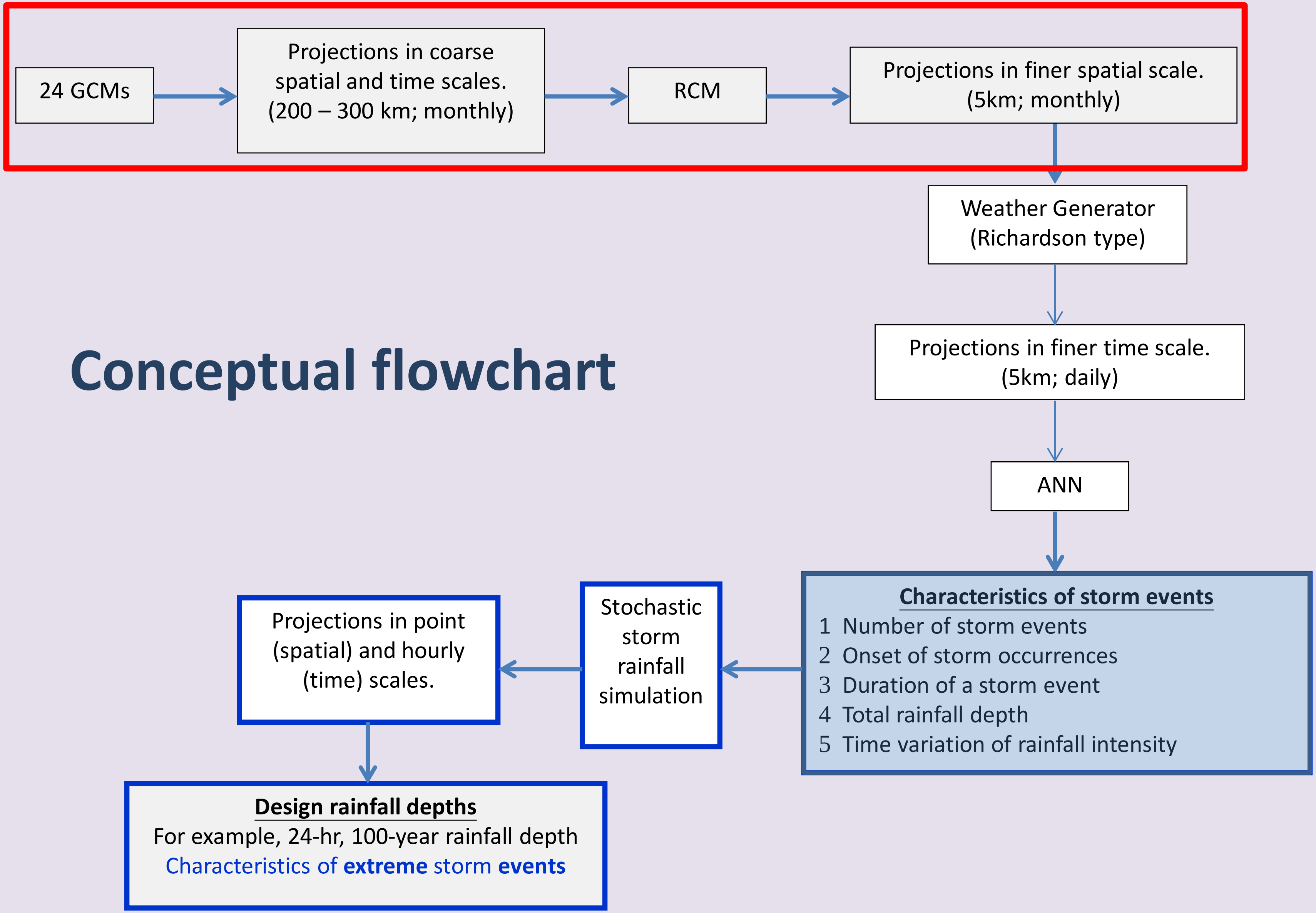
For example, 24-hr, 100-year rainfall depth

Characteristics of **extreme** storm **events**

- **Characteristics of storm events**
    - Number of storm events
    - Duration of a storm event
    - Total rainfall depth
    - Time variation of rainfall intensities
- **These characteristics are random in nature and can be described by certain probability distributions.**
- **Although the realized values of these storm characteristics of individual storm events represent *weather observations*, their probability distributions are *climate (long term and ensemble) properties*.**

- **A GCM – stochastic model integrated approach**
  - **Climatological projection by GCMs**
    - Changes in the means of storm characteristics
    - For examples,
      - Average number of typhoons per year
      - Average duration of typhoons
      - Average event-total rainfall of typhoons
  - **Hydrological projection by a stochastic storm rainfall simulation model**
    - Generating realizations of storm rainfall process using storm characteristics which are representative of the projection period.
    - Preserving statistical properties of the all storm characteristics.

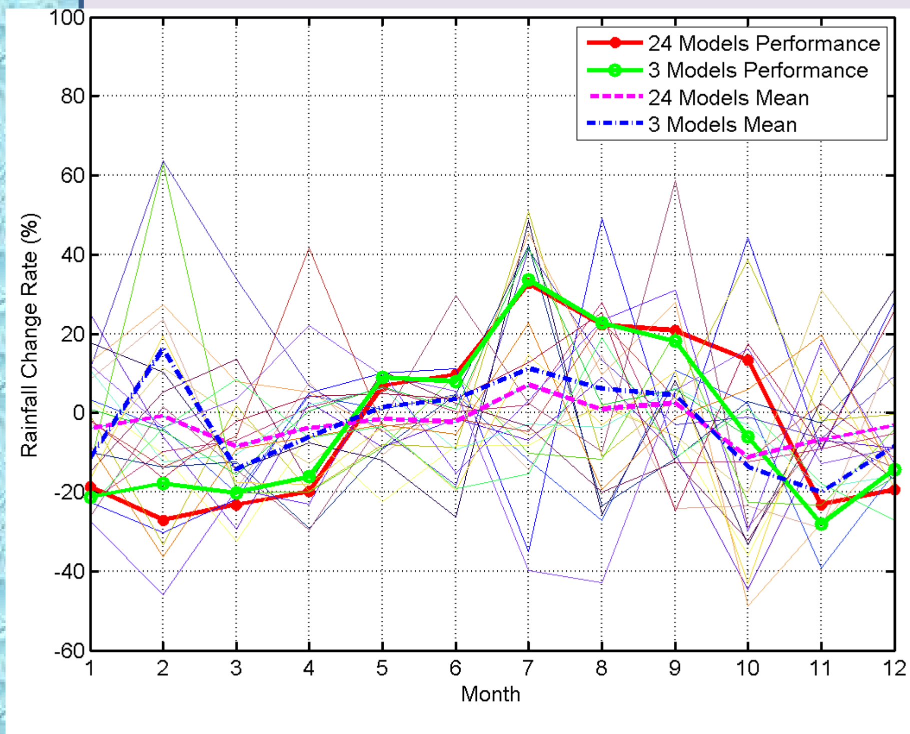
Conceptual flowchart



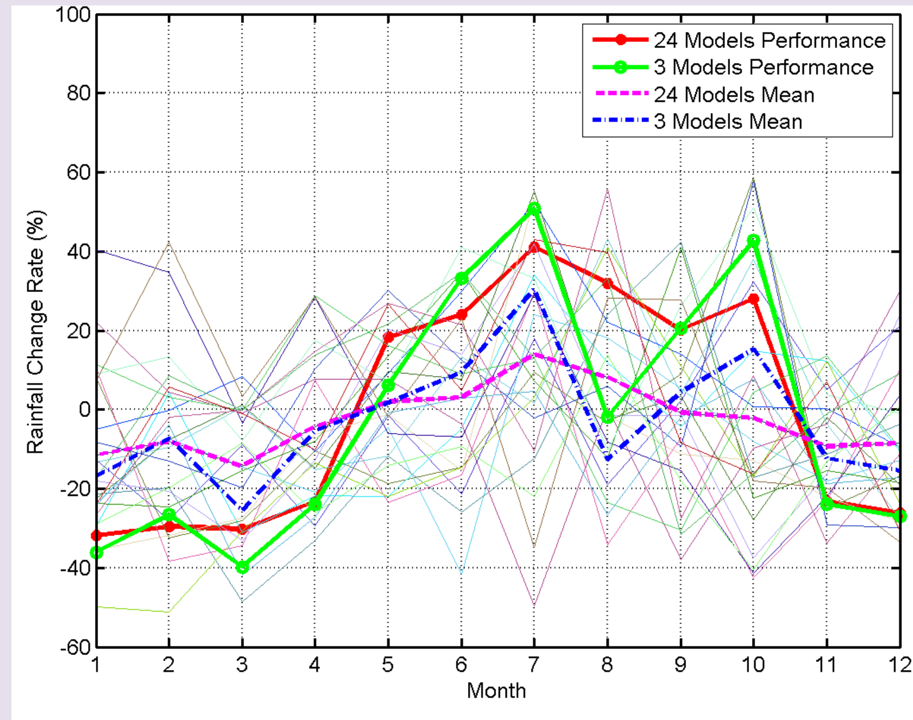
Climate change scenarios and GCM outputs

- Emission scenario: A1B
- Baseline period: 1980 – 1999
- Projection period
  - Near future: 2020 – 2039
  - End of century: 2080 – 2099
- GCM model: 24 GCMs statistical downscaling
- Hydrological scenario: changes in storm characteristics

Changes in monthly rainfalls (Statistical downscaling, Ensemble average with standard deviation adjustment) Taipei area



Near future (2020 – 2039)



Near future (2080 – 2099)

Stochastic Storm Rainfall Simulation Model (SSRSM)

- **Simulating occurrences of storms and their rainfall rates**
  - **Preserving seasonal variation and temporal autocorrelation of rainfall process.**
- **Duration and event-total depth**
- **Inter-event times**
- **Percentage of total rainfalls in individual intervals (Storm hyetographs)**

- **Simulating occurrences of storm events of various storm types**
  - **Number of events per year**
    - Poisson distribution for typhoon and Mei-Yu
  - **Inter-event time**
    - Gamma or log-normal distributions

- **Simulating joint distribution of duration and event-total depth**
  - **Bivariate gamma distribution (e.g. typhoons)**
  - **Log-normal-Gamma bivariate**
  - **Non-Gaussian bivariate distribution was transformed to a corresponding bivariate standard normal distribution with desired correlation matrix.**

$\rho_{XY} \sim \rho_{UV}$ Conversion

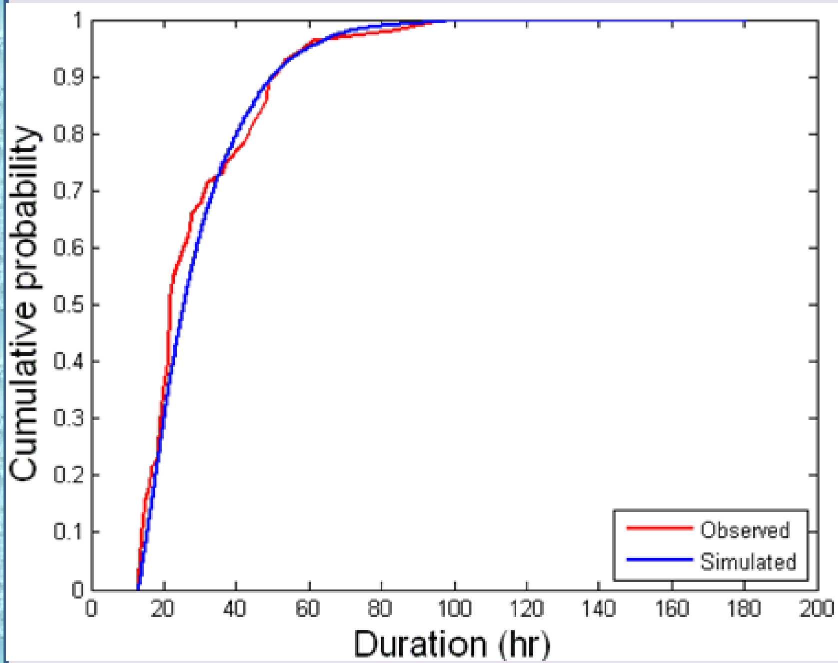
$$\rho_{XY} \approx \left(A_X A_Y - 3A_X C_Y - 3C_X A_Y + 9C_X C_Y\right)\rho_{UV}$$

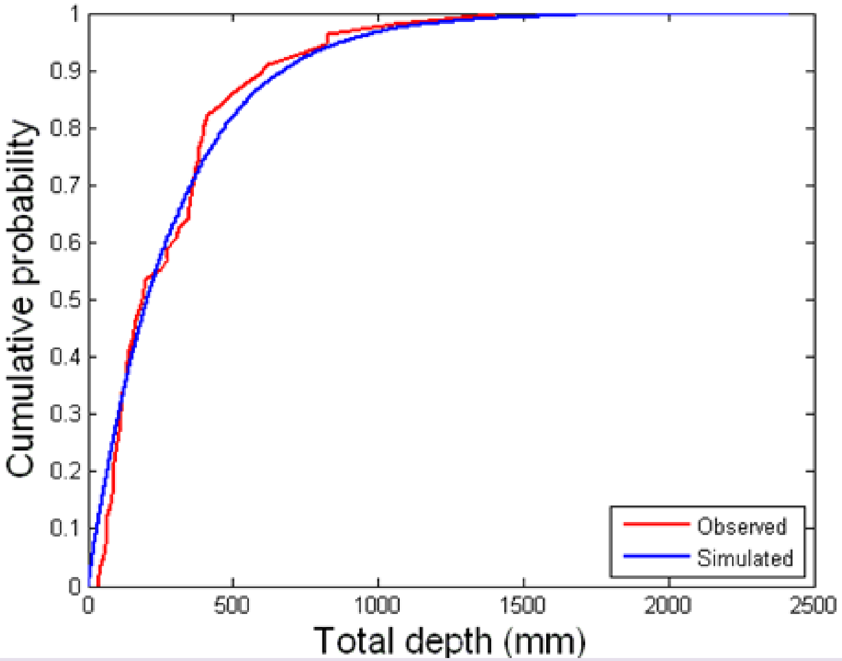
$$+2B_X B_Y \rho_{UV}^2 + 6C_X C_Y \rho_{UV}^3$$

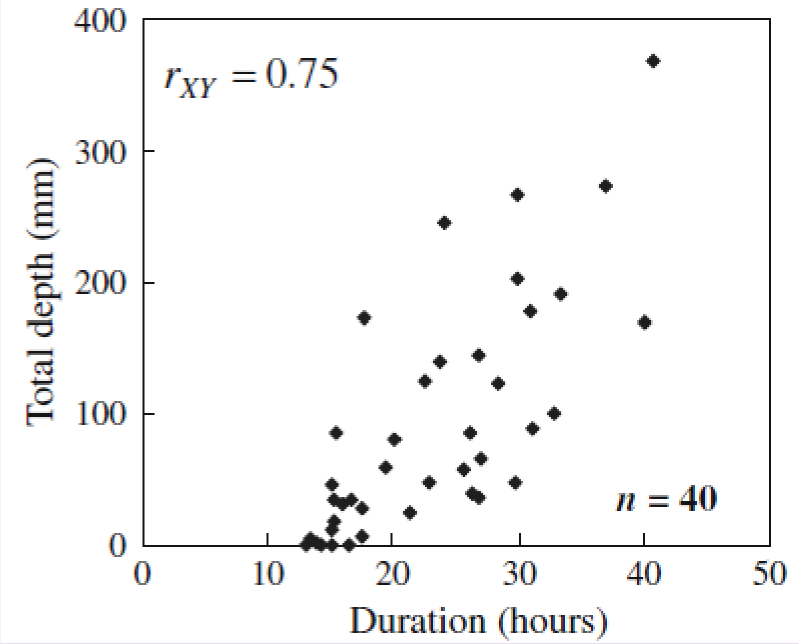
$$A_X = 1 + \left(\frac{\gamma_X}{6}\right)^4 \quad B_X = \frac{\gamma_X}{6} - \left(\frac{\gamma_X}{6}\right)^3 \quad C_X = \frac{1}{3}\left(\frac{\gamma_X}{6}\right)^2$$

$$A_Y = 1 + \left(\frac{\gamma_Y}{6}\right)^4 \quad B_Y = \frac{\gamma_Y}{6} - \left(\frac{\gamma_Y}{6}\right)^3 \quad C_Y = \frac{1}{3}\left(\frac{\gamma_Y}{6}\right)^2$$

Bivariate gamma (X,Y)

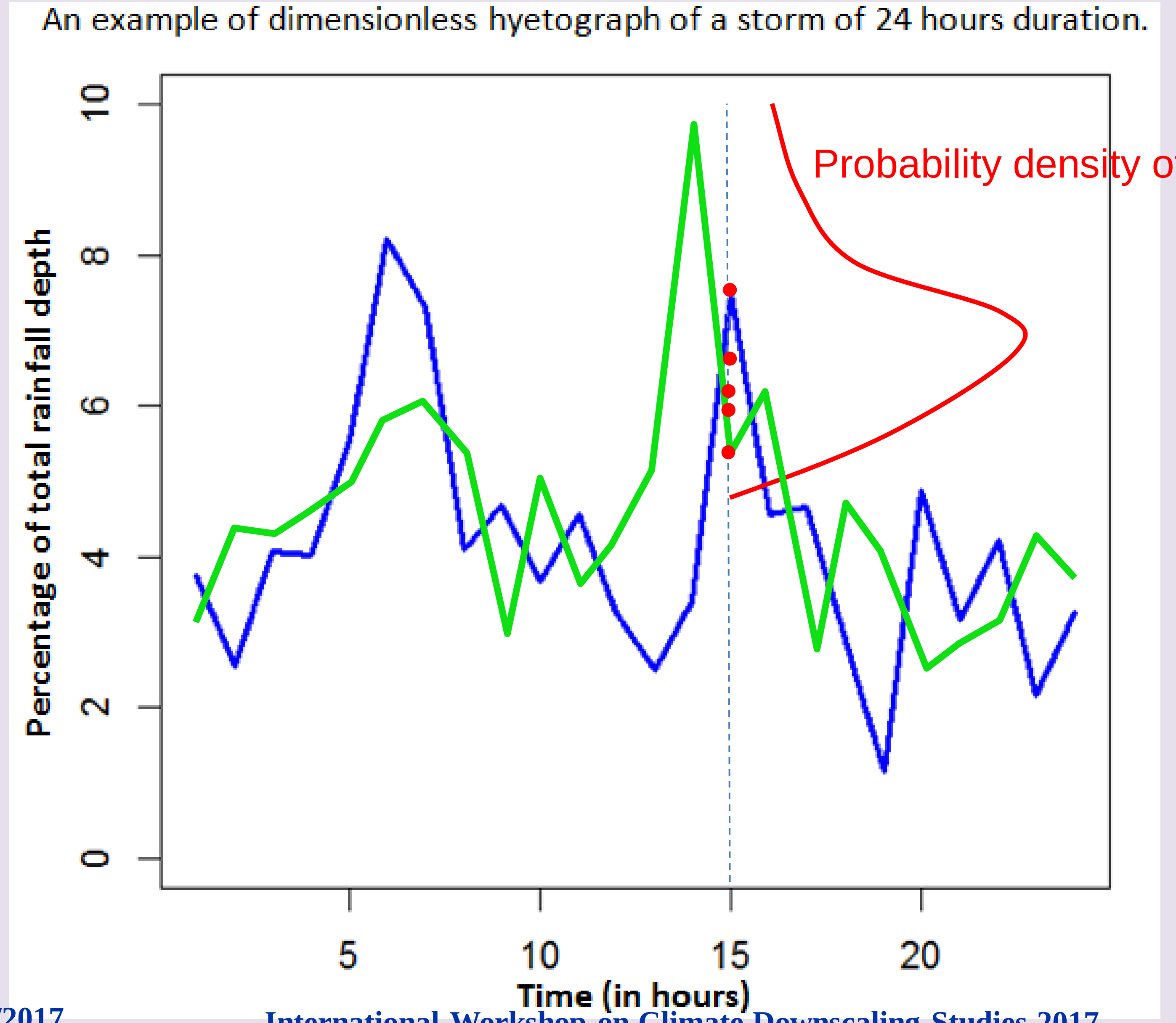

Cumulative probability
Duration (hr)
Observed
Simulated


Cumulative probability
Total depth (mm)
Observed
Simulated


$r_{XY} = 0.75$
$n = 40$
Total depth (mm)
Duration (hours)

- **Simulating percentages of total rainfalls in individual intervals (Simulation of storm hyetographs)**
  - **Based on the simple scaling property**
    - Durations of all events of the same storm types are divided into a fixed number of intervals (e.g. 24 intervals).
    - For a specific interval, rainfall percentages of different events are identically and independently distributed (IID).
    - Rainfall percentages of adjacent intervals are correlated.

Modeling the storm hyetograph



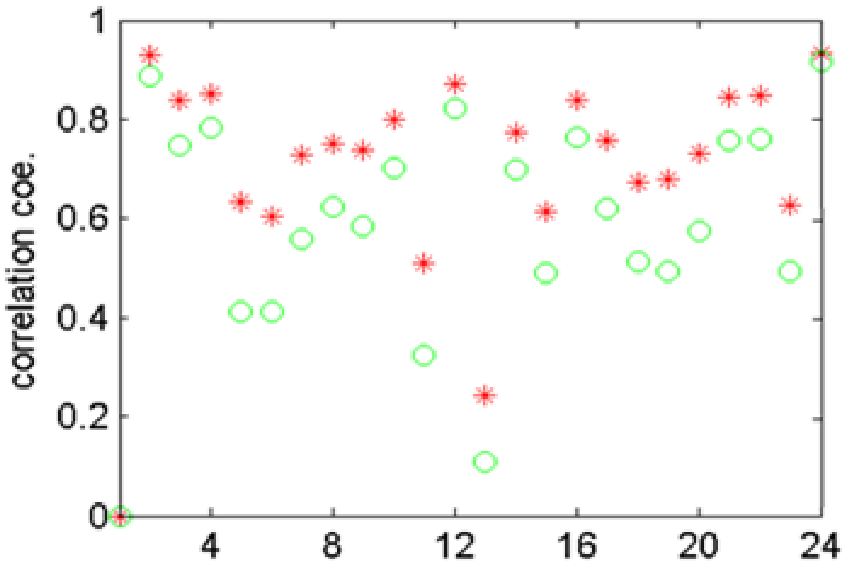
Properties:

1. $0 < x(t) \le 100\%,\ \ t = 1,\ 2,\ \ldots,\ 24.$

2. $X's$ can be described by gamma distributions.

3. $\sum_{t=1}^{24} x(t) = 100$

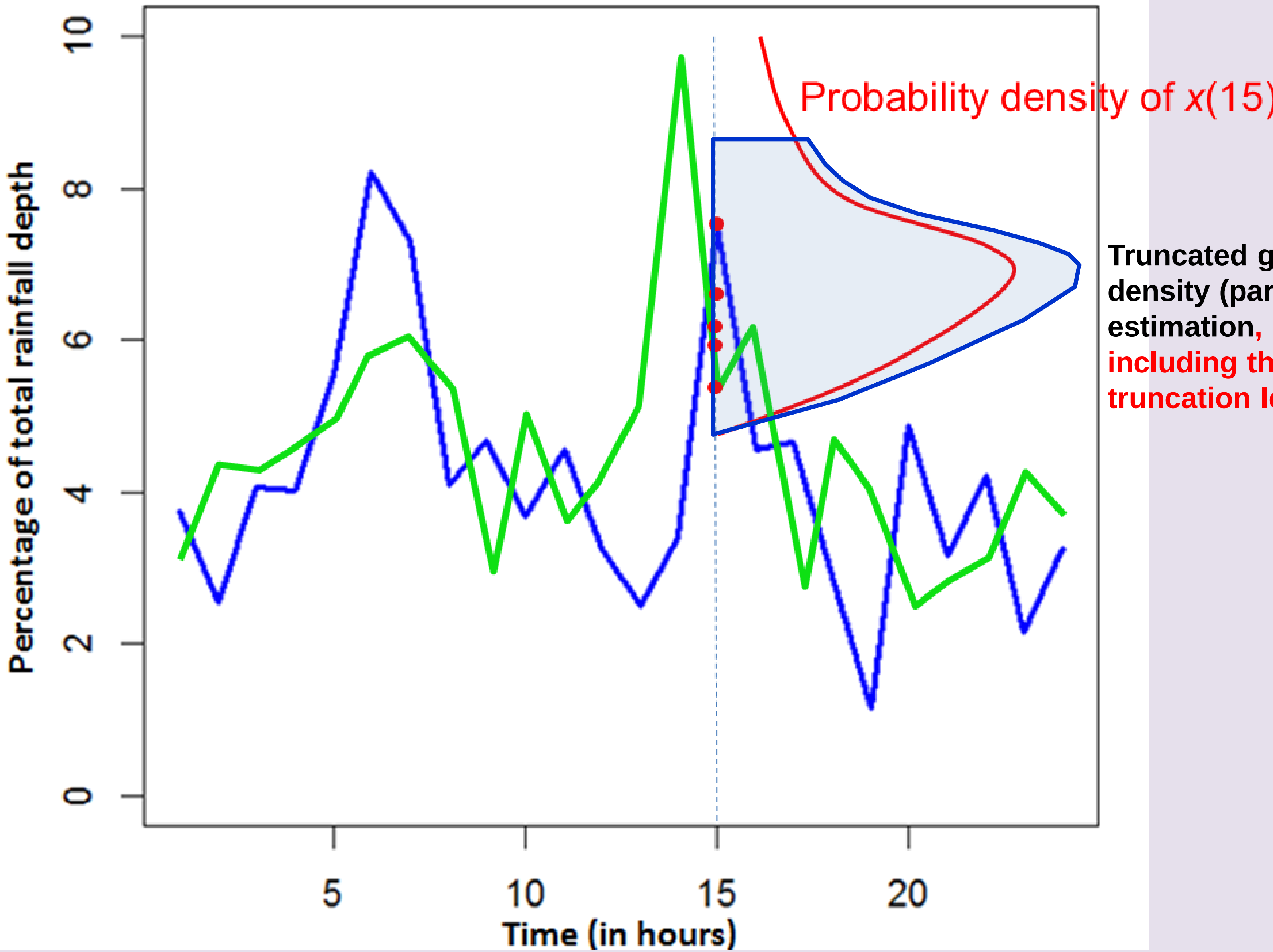
4. Lag-1 autocorrelation coefficients are significantly different from 0. For example, $Correl(x(t), x(t+1)) > 0.5$.



Taking all the above properties into account, we propose to model the dimensionless hyetograph by a <u>truncated gamma Markov process</u>.

5. It is unlikely that rainfall percentage of any particular hour will exceed a level (for example, 30%) which is significantly lower than the ultimate upper level of 100%. Thus, rainfall percentage of any particular hour is modeled as having a truncated gamma distribution (truncated from above).

An example of dimensionless hyetograph of a storm of 24 hours duration.



Truncated gamma density (parameters estimation, including the truncation level)

– **Rainfall percentages should sum to 100%**
  - Truncated gamma distributions
  - Conditional simulation is necessary
  - 1st order Markov process

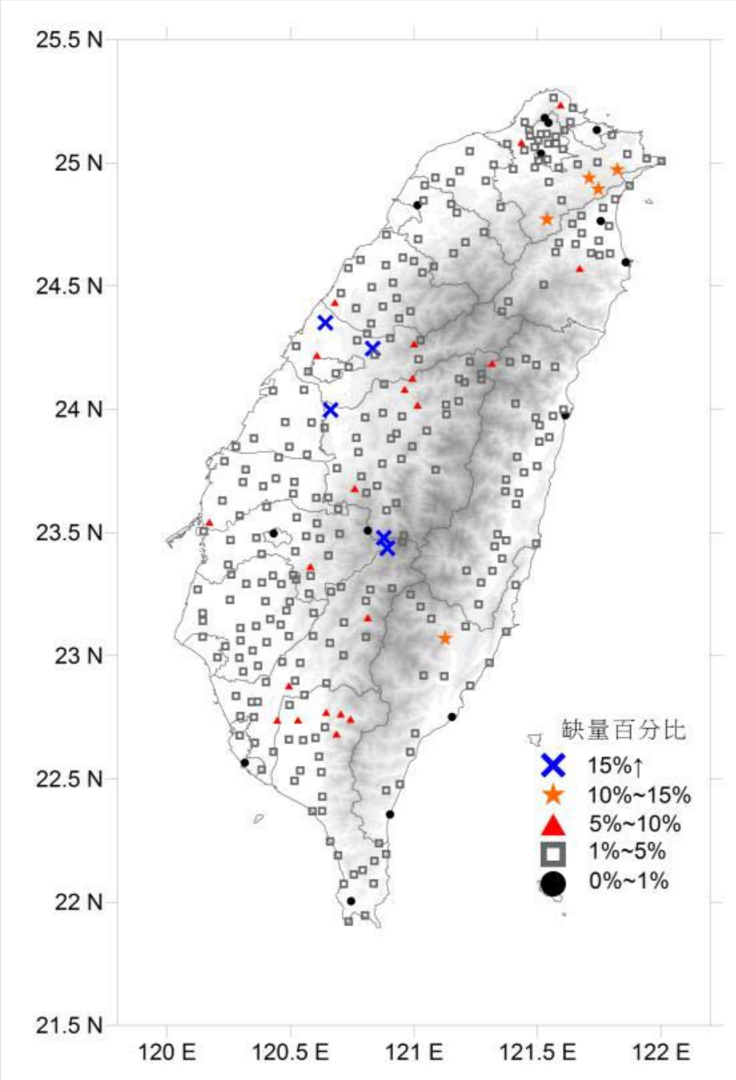
– **Conditional simulation of first order truncated gamma Markov process**

CWB Raingauge Network

More than 400 automated rainfall stations established since early 1990.

Until now, most stations have record length less than or close to 20 years.

If rainfall frequency analysis requires at least 40 years of annual maximum rainfalls, then we have to wait for another 20 years.



Beginning Dates of Some Annual Maximum Events in Taiwan.

	Duration (hours)*									
	1	2	3	4	6	12	18	24	48	72
Year of 1965										
Hosoliau	9/5	8/18	8/18	8/18	8/18	8/18	8/18	8/18	8/18	8/18
Wutuh	6/11	8/21	9/6	9/6	9/6	9/6	8/18	8/18	8/18	8/18
Year of 1969										
Hosoliau	5/14	5/14	5/14	5/14	9/26	9/26	9/9	9/9	9/9	9/9
Wutuh	9/21	9/8	9/8	9/8	9/8	9/8	9/8	9/8	9/8	9/8
Year of 1974										
Hosoliau	9/27	9/27	10/11	10/11	10/11	10/11	10/11	10/11	10/11	10/11
Wutuh	9/15	9/15	10/11	10/11	9/15	10/11	10/11	10/11	10/11	10/11
Year of 1983										
Hosoliau	5/23	5/23	5/23	5/23	5/23	10/10	10/10	10/10	10/10	10/10
Wutuh	10/1	10/1	6.3	6/3	6/3	10/1	10/1	10/1	10/1	10/1
Year of 1987										
Hosoliau	7/26	10/22	10/22	10/22	10/22	10/22	10/22	10/22	10/22	10/22
Wutuh	10/22	7/26	10/22	10/22	10/22	10/22	10/22	10/22	10/22	10/22
Year of 1994										
Hosoliau	6/18	6/18	6/18	6/18	6/18	10/9	10/9	10/9	10/9	10/9
Wutuh	6/18	6/18	6/18	9/12	9/12	9/12	9/12	9/12	9/12	9/12